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Research concentrated on developing probabilistic and statistical methods to solve problems in stochastic networks and system reliability. Six technical reports were written. Titles include: "On Sojourn Time in Jackson Networks of Queues", "The Loop Elimination Algorithm for Inverting Large Sparse Matrices", "Waiting Time and Workload in Queues with Periodic Poisson Input" and "A Reliability Model Based on the Gamma Process and its Analytic Theory".

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Sponsor:	Department of the Air Force Air Force Office of Scientific Research (AFSC) Bolling Air Force Base DC 20332-6448 Major Brian Woodruff, Program Manager
Contractor:	Ford Aerospace & Communications Corporation Western Development Laboratories Division 3939 Fabian Way Palo Alto, CA 94303 Dr. Austin J. Lemoine, Project Director Dr. Michael L. Wenocur, Co-Principal Investigator
Period of Performance:	January 1, 1986 - April 30, 1988 (Contract Signed By AFSC March 12, 1986)

1. INTRODUCTION

Ford Aerospace and Communications Corporation (FACC), Western Development Laboratories Division (WDL) is conducting a comprehensive program of research, sponsored in part by the subject contract, on mathematical modeling. The scientific objectives of this research program are to develop improved probabilistic and statistical methods and to demonstrate their power and relevance through application to specific problems in stochastic networks and system reliability.

2. RESEARCH RESULTS ON PROBABILISTIC METHODS

Research efforts to date of the probabilistic methods program at Ford Aerospace are described in a series of ten technical papers. **The fifth through tenth papers listed below report on research sponsored in part by the subject AFSC Contract.**

1. **"On Failure Modeling,"** A. Lemoine and M. Wenocur.
2. **"A Stochastic Network Formulation for Complex Sequential Processes,"** A. Lemoine.
3. **"On Shot Noise and Reliability Modeling."** A. Lemoine and M. Wenocur.
4. **"Brownian Motion with Quadratic Killing and Some Implications,"** M. Wenocur.
5. **"On Sojourn Time in Jackson Networks of Queues,"** A. Lemoine.
6. **"Diffusion First Passage Times: Approximations and Related Differential Equations,"** M. Wenocur.
7. **"The Loop Elimination Algorithm for Inverting Large Sparse Matrices,"** R. Barack and R. Emberson.
8. **"Approximating Probability Densities on the Positive Half-line,"** M. Wenocur.
9. **"Waiting Time and Workload in Queues with Periodic Poisson Input,"** A. Lemoine.
10. **"A Reliability Model Based on the Gamma Process and its Analytic Theory,"** M. Wenocur.

The papers listed above are now summarized to provide background and perspective on current problems of interest and further areas for investigation.

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2.1 SUMMARY OF RESEARCH REPORTS

The first paper presents a very promising approach to failure modeling which takes account of the dynamic dependency of system failure and decay on the state of the system. Under this approach, system state or wear and tear is modeled by an appropriately chosen random process, eg, a diffusion process; and the occurrences of fatal shocks are modeled by a Poisson process whose rate function is state-dependent. The system is said to fail when either wear and tear accumulates beyond an acceptable or safe level or when a fatal shock occurs. The approach has significant merit. First, it provides new and revealing insights into most of the famous and frequently used lifetime distributions in reliability theory, including the Makeham, Gompertz, Weibull, Rayleigh and Gumbel distributions. In fact, these classic models are obtained in a unified and surprisingly straightforward manner. More significantly, however, the approach suggests intuitively appealing and computationally tractable ways of enhancing these standard failure models and for developing new ones. (*Naval Research Logistics Quarterly, Vol. 32, 1985, pp. 497-508.*)

Stochastic networks, or vector random processes, provide an appealing theoretical framework for modeling complex sequential processes evolving under uncertainty. *The second paper* describes a network formulation for a process wherein an object or "system" moves through a succession of states (nodes) and operating modes (classes) in the course of carrying out its function (fulfilling its purpose). Transitions between states and operating modes occur in a possibly random manner, and require (consume) some resource in randomly varying amounts. The paper discusses the routing behavior and resource requirements of a typical object as it moves through (and eventually out of) the network. We then shift our focus, from a single object and its odyssey, to the network as whole, where time is the resource and many objects are entering the network according to a possibly nonhomogeneous Poisson pattern; in this vein we discuss the evolution of the network over time. Finally, we consider the richness of applications for the formulation and results. (*Naval Research Logistics Quarterly, Vol. 33, 1986, pp. 431-443.*)

The third paper is motivated by the general approach to reliability modeling of the first paper, and explores the implications of shot-noise formulations for system stress. Suppose the system is subjected to shocks or jolts according to a Poisson process with rate λ. Suppose that if a jolt of magnitude D occurs at epoch S then at time $S+t$ the contribution of the jolt to the system stress is $Dh(t)$, where $h(t)$ is a non-negative function, vanishing on the negative half-line. If $\{ S_n, n \geq 1 \}$ are the epochs of shot occurrences and $\{ D_n, n \geq 1 \}$ are the magnitudes of the successive jolts, then the "residual system stress" at time t, say $X(t)$, is given by

$$X(t) = \sum_{n=1}^{\infty} D_n h(t-S_n).$$

Exponential decay of individual shocks corresponds to $h(t) = exp(-at)$. The time to

system failure is taken as the epoch of the first count in a doubly stochastic Poisson process with rate function $X(t)$. This formulation is intuitively appealing and gives tractable results. Indeed, a class of failure distributions is derived using the shot noise model for system stress. The properties of these distributions and their structure are explored. (*Operations Research, Vol. 34, 1986, pp. 320-323.*)

The fourth paper carries forward the general approach to reliability modeling in the first paper. System wear and tear is modeled as a Brownian motion with drift. Failure is due to shock only (the system dies in the line of duty). In state x, shocks occur at rate λx^2. Thus, the time to system failure is the "killing time," say τ, of Brownian motion with drift and quadratic killing rate. The paper derives an explicit formula for the state-dependent survival function of this process, that is, $P^x\{\tau > t\}$. The derivation of this formula is truly stunning, involving the Karhunen-Loeve expansion for Brownian motion, special function theory, and the calculus of residues. Some properties of this killing time distribution are also analyzed. (*Journal of Applied Probability, Vol. 23, 1986, pp. 893-903.*)

The fifth paper is concerned with representations for equilibrium sojourn time distributions in Jackson networks of queues. For a network with N single-server nodes let h_i be the Laplace transform of the residual system sojourn time for a customer "arriving" to node i, "arrival" meaning external input or internal transfer. The transforms $\{h_i : i=1,...,N\}$ are shown to satisfy a system of equations we call the *network flow equations*. These equations also lead to a general recursive representation for the higher moments of the sojourn time variables $\{T_i : i=1,..N\}$. Combining these formulas with an appropriate martingale, explicit expressions are obtained for the second moment of sojourn time, and the covariance of sojourn time and queue size found upon "arrival," in single-server Markovian queues with feedback; the expression for second moment of sojourn time derived here coincides with the result reported by Takacs in 1963. The paper employs probabilistic methods to obtain probabilistic results. These results and the methods employed hold promise for making further breakthroughs on an important problem which has remained remarkably resistant to analysis. (*Journal of Applied Probability, Vol. 24, 1987, pp. 495-510.*)

The sixth paper is concerned with practical and theoretical considerations in computing first passage time statistics. A finite spectral expansion employing the method of finite moments is derived for the first passage distribution and is shown to converge at nearly factorial rates. The paper is motivated by first passage times as models of failure times. Indeed, this paper continues the line of development initiated in the first paper, where a stochastic process is used to model system state, ie, *wear-and-tear*, and the system fails when either a traumatic killing event occurs (killing events happen with rate $k(x)$ in state x), or the system is retired when *wear-and-tear* reaches some predefined threshold (ie, a first passage occurs). For example, if system state is modeled as Brownian motion with positive drift, then first passage

to a specified threshold has an inverse Gaussian distribution. This first passage distribution has been successfully applied to numerous problems to obtain good fits. A related but parallel line of development is explored in the fourth paper, where the killing time distribution of Brownian motion with quadratic killing rate is calculated. The aim of this sixth paper is to study first passage time distributions, where the system state process is a general diffusion with reflection at the origin and absorption at $r < \infty$. That is, the system state evolves as a diffusion, and failure occurs at the epoch of first passage (or absorption) to level r . (*Stochastic Processes and Their Applications, Vol. 27, 1988, pp. 159-177.*)

The seventh paper presents a promising method for inverting large sparse matrices of form $I-P$, where I is the identity matrix and P is nonnegative. For P sub-stochastic, matrices of this form are basic to Markovian queueing networks, which represent a fundamental approach to modeling and performance evaluation of large, distributed computer/communications systems. The inversion technique of this paper, called the Loop Elimination Algorithm, has been developed and tested extensively. The Loop Elimination Algorithm is quite general, however, and should find application to flow problems in large networks. (To be submitted for publication.)

The eighth paper presents and discusses a variety of methods for approximating probability density functions on the positive half-line. The paper focuses on the class of approximants having rational Laplace transforms. It is well known that such approximation is possible, but explicit methods for obtaining approximations have not been extensively studied. A survey of appropriate numerical analysis techniques is provided, with some new twists relevant to the goal of approximating probability densities. In particular, the method of moments and orthogonal expansion methods are studied. A new proof is given that continuous probability densities vanishing at ∞ can be uniformly approximated by generalized hyper-exponential densities. The same denseness property is also shown to hold for families of densities expressible as sums of Erlang densities *with common fixed rate parameter.* (Submitted for publication.)

The ninth paper carries forward the line of development for queues with periodic Poisson input initiated in Harrison and Lemoine [4] and continued in Lemoine [10]. This paper signals a breakthrough on a difficult problem in the field, which has attracted considerable interest. Formulas are obtained for asymptotic workload and waiting time distributions. These formulas involve the corresponding moments of waiting-time (workload) for the M/G/1 system with the same average arrival rate and service distribution. In certain cases, all the terms in the formulas can be computed exactly, including moments of workload at each 'time of day.' The approach makes use of an asymptotic version of the Takacs integro-differential equation for the time-dependent distribution of workload state, together with results from [4] and [10]. (To appear in *Journal of Applied Probability.*)

The tenth paper studies reliability models using the gamma process as a stress driver, continuing a line of research first suggested in Lemoine and Wenocur [11], [12]. A reliability model is developed where system state is a random process satisfying a stochastic differential equation in which the driving process is gamma distributed. The necessary Markov process analytic theory is developed. In particular, the infinitesimal generator for the state process is obtained, and an integro-differential equation for Feynman-Kac functionals is derived and shown to uniquely determine the functional. (Submitted for publication.)

3. DIRECTIONS FOR FURTHER RESEARCH ON PROBABILISTIC METHODS

Directions for further research on probabilistic methods prompted by our results and suggested by preliminary exploration, include the following:

3.1 PARAMETER ESTIMATION AND DATA ANALYSIS

The state-dependent approach of Lemoine and Wenocur [11] to modeling system failures presents both significant challenges and opportunities for statistical analysis. In order to apply this approach, one must estimate both state-process parameters and the killing rate function. In most reliability modeling efforts there will be a natural candidate for system state, and the corresponding state process will be observable. The observability permits the study of conditional probability of failure given the state process. Indeed, the conditional distribution of failure due to trauma is simply the distribution of the first event in a nonhomogeneous Poisson process whose rate at time t is equal to $k(X(t))$, where $X(\cdot)$ is the system state process and $k(\cdot)$ is the killing rate function. It is this fact that allows the analyst to decompose the parameter estimation problem into two distinct and loosely coupled statistical estimation problems; namely, the estimation of the killing rate function and the estimation of the system state process parameters.

Furthermore, with the evolution of micro-electronics hardware, there is an ever increasing system self-monitoring capability. The benefits of self-monitoring can only be properly exploited by using state dependent failure modeling. A great deal can be accomplished using data collected by self-monitoring systems. It can be used to improve understanding of failure as a function of system state. Such understanding can result in better methods for early failure prediction. An important first step in developing the mathematical techniques needed to exploit monitoring information is embodied by the dynamic failure modeling approach suggested in [11].

We plan to explore estimation problems suggested by the state-dependent approach to modeling system failures, that is, estimation of state-process parameters and killing-rate function. A starting point is the literature on statistical inference for stochastic processes, cf. Basawa and Prakasa Rao [2], Kutoyants [9], and Nanthi [18]. We anticipate that methods of conditional maximum likelihood, cf. Cox and Hinkley [3]; survival analysis, cf. Miller [17]; multiplicative intensity models for multivariate counting processes, cf. Jacobsen [5]; point processes, cf. Karr [7]; and martingales and stochastic integrals, cf. Metivier and Pellaumail [15], Metivier [16], and Karatzas and Shreve [6], will be useful.

3.2 SOJOURN TIME IN NETWORKS OF QUEUES

Based on the results of Lemoine [13], we envision several promising approaches to approximating the higher moments of sojourn times in Jackson networks of queues. In particular we have derived a functional equation which permits us to approximate the second moments. This functional equation has some intriguing properties, eg, it is a positive linear map from $l_1(\pi)$ sequences to $l_1(\pi)$, indeed a contraction mapping possessing a unique solution. We are cautiously optimistic that this approach could lead to a closed form solution for the second moments of sojourn times.

We believe also that martingale methods and time reversal combined with our representation results for stochastic networks could provide the means to obtain approximations (or exact solutions) to higher moment problems and Laplace transforms.

3.3 MOMENT CONSTRAINED ADAPTIVE METHODS FOR ORTHOGONAL EXPANSIONS

This method, suggested in Section 5 of Wenocur [21], holds great promise for improving the accuracy of probability density approximation on the positive half-line. Before this method can produce useful results, a number of important issues must be investigated. Namely, find conditions under which the supremum is obtainable and unique; identify what special structure may be exploited to obtain efficient algorithms; and how stable and computationally expensive this approach may be.

We envision this task will require both theoretical and computational efforts.

3.4 IMPROVED BERNSTEIN TYPE APPROXIMATIONS

Bernstein polynomial approximation is an important method for both theoretical and numerical analysis. Bernstein polynomial approximation being a positive operator makes it very attractive as a technique for probability density approximation (cf. Section 2 of Wenocur [21]). However, its slow rate of convergence is a significant drawback in this context. A theorem of Korovkin (cf. Chapter 4 of [8]) states that positive linear operators can achieve n^{-2} rate of convergence but no more. In contrast, standard Bernstein polynomials achieve only n^{-1} rate of convergence .

Our goal is to produce an approximation method similar to Bernstein approximation, but obtaining the optimal n^{-2} optimal rate. Our preliminary study has produced a method having $n^{-5/4}log(n)$ rate of convergence.

3.5 IMPROVED DIFFUSION APPROXIMATIONS FOR BIRTH-DEATH MARKOV CHAINS

Preliminary results strongly suggest that alternate parametrization may significantly improve accuracy of diffusion approximations for M/M/s under light or heavy traffic conditions when compared to the standard approach.

3.6 RELIABILITY MODELS BASED ON DIFFUSION AND OTHER MARKOV PROCESSES

We intend to carry forward the line of development for dynamic reliability modeling initiated in Lemoine and Wenocur [11] and continued in Wenocur [19], [22]. In particular, we will extend the results of Wenocur [19] to the setting where the Ornstein-Uhlenbeck process is the model for system stress and traumatic killing events are a quadratic function of system state. We will also investigate theoretical and computational aspects of the Markov analytic theory developed in [22].

3.7 QUEUES WITH PERIODIC POISSON INPUT

As shown in [14], exact computation of moments for asymptotic workload and waiting time distributions involves the values $\{H_s(0)\}$ where $H_s(0)$ is the long-run probability that server workload is zero at 'time of day s.' When these probabilities are known the moments can be calculated explicitly. If the service distribution is concentrated on integer multiples of the period length, these probabilities are equal (over the period) and known. Our objective is to compute (or approximate) the values $\{H_s(0)\}$ for arbitrary distributions of service time, and to explore implications for calculating (or estimating) asymptotic distributions for workload and waiting time.

3.8 INVERTING LARGE SPARSE MATRICES

Our goal is to characterize the class of matrices for which the Loop Elimination Algorithm [1] converges, and/or find an efficient method for checking same. (An appropriate analogy is cycling in the simplex method.)

4. RELATED CONTRACT ACTIVITY

Supported in part by the subject AFSC contract, A. Lemoine and M. Wenocur attended the following professional meeting:

- Special Interest Conference, "Queueing Networks and Their Applications." This conference was held at the Hyatt Regency Hotel in New Brunswick, New Jersey, January 7-9, 1987.
- Sixteenth (16th) Conference on Stochastic Processes and their Applications. This conference was held at Stanford University, August 16-21, 1987.

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